Robotic Arm

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Project Description

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- Stroke is the leading cause of upper limb disability.
- Survivors often lose mobility in one arm, limiting daily activities.
- Goal: develop a **waist-mounted robotic arm** that
    - Offers **active gravity compensation**
    - Remains **lightweight, low-profile, and energy efficient**
- Enables the arm to rest naturally by the user's side.
- **Client:** Dr. Zach Lerner, Associate Professor of Mechanical Engineering, NAU.
- **Sponsorship:** W.L Gore

Background & Benchmarking

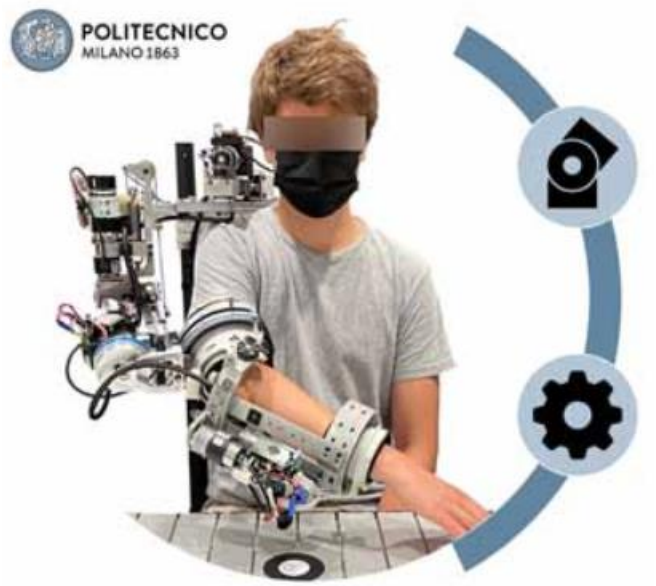

Figure 1: Agree Exo Exoskelton [1]

Arm Exoskeleton uses spring-pulley antigravity system to minimize torque requirements

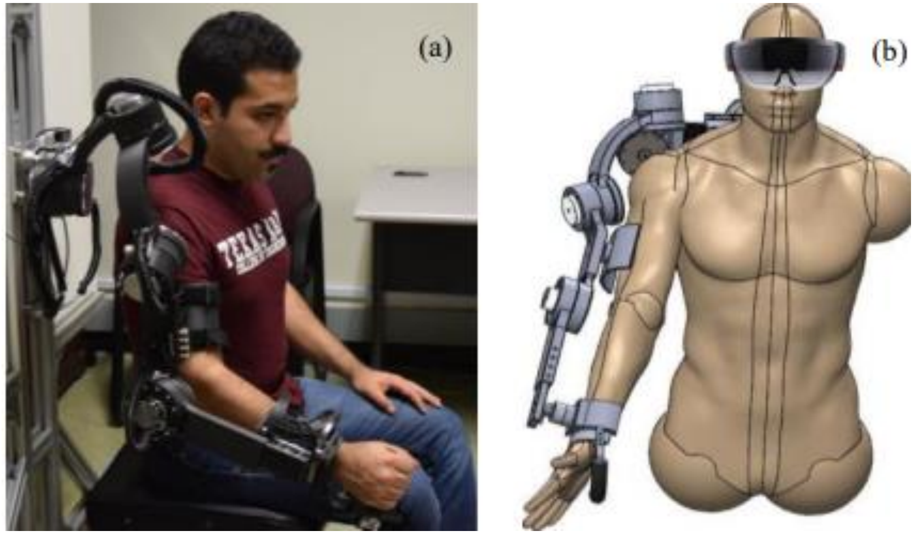

Figure 2: CleverArm Exoskeleton [2]

Has a focus in compactness as well as having 8 degrees of freedom

Figure 3: ExoFlex Exoskeleton [3]

A hybrid exoskeleton with both rigid and soft components making up the device

Customer/Engineering Requirements

- Customer Requirements
  - Comfortability
  - Range of Motion
  - Safety
  - Cost for Consumer
  - Durability
  - Ease-of-Use
  - Low-Profile

- Engineering Requirements
  - Degrees of Freedom (>4)
  - Quality of Components (< $1400)
  - Quality of Materials (< $1000)
  - Manufacturing Cost (< $1000)
  - Torque Speed (60 °/s)
  - Battery Life (> 8 Hours)
  - Weight (< 2 kg)

QFD

	Degrees of Freedom	Quality of Components	Quality of Materials	Manufacturing Cost	Torque Speed	Battery Life
Degrees of Freedom						
Quality of Components	pos					
Quality of Materials		neg				
Manufacturing Cost	pos		pos			
Torque Speed		pos				
Battery Life	neg	pos		neg	pos	
Weight	pos	pos	pos	pos	pos	pos

Correlation	
Positive	pos
Negative	neg

Relationship	
Strong	9
Moderate	3
Weak	1

AgreeExo	A
ExoFlex	B
CleverArm	C

			Engineering Requirements							Benchmarking		
Relative Weight (%)	Customer Weights	Customer Requirments	Degrees of Freedom	Quality of Components	Quality of Materials	Manufacturing Cost	Torque Speed	Battery Life	Weight	Poor	Adequate	Excellent
11	4	Comfortable	3	3	3	1	3	3	9		C	AB
22	5	Range of Motion	9	3	3	1	1	1	1	A	B	C
10	5	Safety	3	9	9	1	9	1	3			ABC
10	2	Cost for Consumer	3	9	9	9	3	3	3	AC	B	
5	3	Durabiltiy	1	9	9	1	1	1	9		B	AC
5	4	Ease-of-use	9	3	3	1	9	3	3	BC	A	
22	3	Low-Profile	3	1	3	3	3	9	9	ABC		
		Technical Requirement Units	N/A	$	$	$	°/s	Hours	kgs			
		Technical Requirement Targets	4	<1400	<1000	<1000	60	>8	<2			

Literature Review

- [4] M. E. Moran, "Evolution of robotic arms," *Journal of Robotic Surgery*, vol. 1, no. 2, pp. 103–111, May 2007, doi: https://doi.org/10.1007/s11701-006-0002-x.

  It is a book about the evolution of robotic arms throughout history.

- [5] B.-O. Lee, Ita Daryanti Saragih, and Sakti Oktaria Batubara, "Robotic arm use for upper limb rehabilitation after stroke: A systematic review and meta-analysis," vol. 39, no. 5, pp. 435–445, Mar. 2023, doi: https://doi.org/10.1002/kjm2.12679.

  It is a compilation of several studies done for robotic arms doing rehabilitation in the upper limbs after a stroke.

- [6] ISO, *ISO/TS 15066:2016 - Robots and robotic devices — Collaborative robots*, 2016. [Online]. Available: https://www.iso.org/standard/62996.html

  Safety Standards for human robotic interaction in human robotics.

- [7]A. Kojima, D. T. Tran, and J.-H. Lee, "Investigation of the Mounting Position of a Wearable Robot Arm," *Robotics*, vol. 11, no. 1, p. 19, Feb. 2022, doi: https://doi.org/10.3390/robotics11010019.

  Information about mounting positions that can help design process.

Literature Review

- [8] M. Y. Metwly, C. L. Clark, J. He, and B. Xie, “A Review of Robotic Arm Joint Motors and Online Health Monitoring Techniques,” *IEEE Access*, pp. 1–1, Jan. 2024, doi: https://doi.org/10.1109/access.2024.3447573.

Review of motors used in robotic arms to help part selection.

- [9] S. Parasuraman, Kee Chew Yee, and Arif Oyong, “Human upper limb and arm kinematics for robot based rehabilitation,” Jul. 2009, doi: https://doi.org/10.1109/aim.2009.5229906.

Information of the kinematics of arm movement and how it can be used for robotics.

- [10] K.-H. Cho and W.-K. Song, “Effects of two different robot-assisted arm training on upper limb motor function and kinematics in chronic stroke survivors: A randomized controlled trial,” *Topics in Stroke Rehabilitation*, pp. 1–10, Aug. 2020, doi: https://doi.org/10.1080/10749357.2020.1804699.

Background information that can be used to show the effectiveness of robotic arms used in the rehabilitation of stroke survivors.

Literature Review

- [11] Y. Wang, W. Li, S. Togo, H. Yokoi, and Y. Jiang, “Survey on Main Drive Methods Used in Humanoid Robotic Upper Limbs,” Cyborg and Bionic Systems, vol. 2021, pp. 1–12, Jun. 2021, doi: https://doi.org/10.34133/2021/9817487.

  Addresses the strengths and weaknesses of different driving mechanisms for robotic upper limbs.

- [12] E. Bardi, M. Gandolla, F. Braghin, F. Resta, A. L. G. Pedrocchi, and E. Ambrosini, “Upper limb soft robotic wearable devices: a systematic review,” Journal of NeuroEngineering and Rehabilitation, vol. 19, no. 1, Aug. 2022, doi: https://doi.org/10.1186/s12984-022-01065-9.

  Explains current approaches for designing upper limb exosuits.

- [13] D. Verdel, S. Bastide, N. Vignais, O. Bruneau, and B. Berret, “Human Weight Compensation With a Backdrivable Upper-Limb Exoskeleton: Identification and Control,” Frontiers in Bioengineering and Biotechnology, vol. 9, Jan. 2022, doi: https://doi.org/10.3389/fbioe.2021.796864.

  Discusses driving mechanisms specifically for supporting the weight of an arm.

- [14] H. Zheng et al., “3D Printing Continuous Fiber Reinforced Polymers: A Review of Material Selection, Process, and Mechanics-Function Integration for Targeted Applications,” Polymers, vol. 17, no. 12, pp. 1601–1601, Jun. 2025, doi: https://doi.org/10.3390/polym17121601.

  Explains different fibers and matrices that can be printed for manufacturing versatile parts.

Literature Review

- [15] ISO, "Robots and robotic devices — Safety requirements for personal care robots," ISO 13482:2014, International Organization for Standardization, Geneva, Switzerland, 2014. [Online]. Available: https://www.iso.org/standard/53820.html

Defines safety requirements for “personal care robots,” including “physical assistant robots” (which exoskeletons / assistive arms often are).

- [16] Q. Fang, G. Li, T. Xu, J. Zhao, H. Cai, and Y. Zhu, "A Simplified Inverse Dynamics Modelling Method for a Novel Rehabilitation Exoskeleton with Parallel Joints and Its Application to Trajectory Tracking," Mathematical Problems in Engineering, vol. 2019, Article ID 4602035, 10 pages, 2019, doi: 10.1155/2019/4602035. [Online]. Available: https://pdfs.semanticscholar.org/637d/9c65e0053fe6b7fb3adddf370617520c7cd3.pdf

Includes inverse dynamics for an exo + human arm model, to compute required torques at shoulder & elbow given arm plus exo inertia & motion profiles.

- [17] P. Raghavan, “Upper Limb Motor Impairment After Stroke,” *Physical Medicine and Rehabilitation Clinics of North America*, vol. 26, no. 4, pp. 599–610, Nov. 2015, doi: https://doi.org/10.1016/j.pmr.2015.06.008

Overview of types of impairment: weakness/paralysis, sensory loss, learned non-use, abnormal synergies, etc.

Literature Review

- [18] S. Openshaw, E. Taylor, G. Minder, W. Witherow, T. J. Long, and M. Ford, *Ergonomics and Design: A Reference Guide*, Allsteel Inc., Muscatine, IA, April 2006. doi: https://ehs.oregonstate.edu/sites/ehs.oregonstate.edu/files/pdf/ergo/ergonomicsanddesignreferenceguidewhitepaper.pdf Environmental Health and Safety

  This design guide for ergonomics highlights the importance of strike zones and the limits of the human body.

- [19] G. R. Ballee, *Industrial Maintenance and Mechatronics*, 2nd ed., Goodheart-Willcox Company, Inc., Tinley Park, IL, 2022. ISBN: 978-1637767115. [Chapter 12]. doi: G-W+1

  Chapter 12 is all about belt drives and gear drives, very useful when determining the drivers behind our motor system.

- [20] "Understanding Steel Tube and Pipe Metallurgy," *The Fabricator*, Jul. 18, 2019. doi: https://www.thefabricator.com/tubepipejournal/article/tubepipeproduction/understanding-steel-tube-and-pipe-metallurgy The Fabricator

  This source provides insight into the strength of tubing and how to select the right material for our desired applications.

- [21] R. T. Barrett, *Fastener Design Manual*, NASA Reference Publication 1228, Lewis Research Center, Cleveland, OH, USA, Mar. 1990. doi: https://ntrs.nasa.gov/api/citations/19900009424/downloads/19900009424.pdf NASA Technical Reports Server

  This handbook from NASA captures how to design for the use of fasteners in confined or small spaces/applications.

Literature Review

- [22] *Mechanisms*, Lunyax (Wordpress.com), Jun. 2014. doi: https://lunyax.wordpress.com/wp-content/uploads/2014/06/mechanisms.pdf lunyax's Blog

Handbook on mechanism, but more importantly, on the Degrees of Freedom which we will be heavily invested in during this project.

- [23] *Handbook on Polymer Selection for Engineering*, Google Books. doi: https://books.google.com/books?hl=en&lr=&id=-MB2DhLQ-q0C&oi=fnd&pg=PA421&dq=handbook+on+polymer+selection+for+engineering … Google Books

Polymer selections, something we should have a source for if we use polymers. We likely will, as polymers are widely used in biomedical applications, and we could use their high strength and low weight properties in our own design.

- [24] "Metal Strength Chart - A Pro Guide 2025," PartMFG, 2025. doi: https://www.partmfg.com/metal-strength-chart-a-pro-guide-2025/ partmfg.com

A quick guide on metal strength, from hardness to stresses. Useful for our project, whether it is for gear design or exoskeletal application.

- [25] "ASME Y14.5 GDT Standard," GD&T Basics. [Online]. Available: https://www.gdandtbasics.com/asme-y14-5-gdt-standard/

The CAD drafting standard for all drawings with GD&T. This will be used ot ensure the drawingas we send out for machining are up to standard.

Literature Review

- [26] X. Fan et al., A humanoid robot teleoperation approach based on waist–arm coordination | emerald insight, doi: https://www.emerald.com/insight/content/doi/10.1108/IR-12-2022-0306/full/html

  Different methods used waist-arm coordination (WAC) and dual-arm coordination (DAC), just to get an idea of the project

- [27] K. Kruthika, B. M. K. Kumar, and S. Lakshminarayanan, "Design and development of a robotic arm | IEEE conference publication | IEEE xplore," IEEEXplore, doi: https://ieeexplore.ieee.org/abstract/document/8053274/

  Although not attached to hip, this may explain how a robotic arm functions and the basics.

- [28] C. Ochieze, S. Zare, and Y. Sun, "IOPscience," Progress in Biomedical Engineering, doi: https://iopscience.iop.org/article/10.1088/2516-1091/acc70a/meta?utm_source

  Looks at biomedical engineering and the systems that they are working on. Included rigid-joint exoskeletons, soft exoskeletons, and focused on medical devices used for human rehabilitation.

- [29] M. A. Gull, S. Bai, and T. Bak, "A review on design of upper limb exoskeletons," MDPI, doi: https://www.mdpi.com/2218-6581/9/1/16?utm_source

  Reviews more basic knowledge about exoskeletons being used for the upper body, mainly for the arm.

Literature Review

- [30] Comparison of material's properties for exoskeletons structure | downloaded scientific diagram, doi: https://www.researchgate.net/figure/Comparison-of-materials-properties-for-exoskeletons-structure_tbl4_365212324

  Provides a list of materials used in an exoskeleton and provides important information (pros, cons)

- [31] Z.-J. Chen, "Exoskeleton-assisted anthropomorphic movement training for the upper limb after stroke: The EAMT randomized trial | stroke," AHAjournals, doi: https://www.ahajournals.org/doi/10.1161/STROKEAHA.122.041480

Compares the effects of exoskeletons and physical therapy among stroke patients. May further our understanding of the mobility of different stroke patients.

- [32] Y. Zhao, H. Wu, M. Zhang, J. Mao, and M. Todoh, "Design methodology of portable upper limb exoskeletons for people with strokes," Frontiers, doi: https://www.frontiersin.org/journals/neuroscience/articles/10.3389/fnins.2023.1128332/full?utm_source

This provides the mobility and portability of an exoskeleton device, along with its uses and what it can do for stroke patients. Includes all the components of what methods may be used, like materials, actuation, and what motors may be used.

- [33] Y.-S. Li-Baboud et al., "Evaluation methods and measurement challenges for industrial exoskeletons," MDPI, https://www.mdpi.com/1424-8220/23/12/5604

  Includes tables that go over the functional, ergonomic, and task performance metrics. Also, current and proposed exoskeleton standards.

Literature Review

- [34] G. A. Pratt and M. M. Williamson, “Series elastic actuators,” in *Proc. IEEE/RSJ Int. Conf. Intelligent Robots and Systems (IROS)*, 1995, pp. 399–406, doi: https://doi.org/10.1109/IROS.1995.525827

Discusses the need for Powered exo skeletons to require lightweight and efficient actuators in order for accurate torque control.

- [35] P. Yu, W. Chen, and C. Liu, “Quasi-direct-drive actuation for shoulder exoskeletons: Design and evaluation,” *IEEE/ASME Trans. Mechatronics*, vol. 25, no. 5, pp. 2410–2421, 2020, doi: https://doi.org/10.1109/TMECH.2020.2995134

Discusses importance of high-performance actuators, and the need to compromise bandwidth to improve compliance for actuation in a shoulder exoskeleton.

- [36] E. Moubarak, K. Pham, R. Moreau, and E. Redarce, “Gravity compensation of an upper extremity exoskeleton,” in *Proc. IEEE EMBC*, 2010, pp. 4479–4482, doi: https://doi.org/10.1109/IEMBS.2010.5626036

Passive vs active gravity compensation methods: good place to start with initial design decisions.

- [37] F. Just *et al.*, “Human arm weight compensation in rehabilitation robotics: efficacy of three distinct methods,” *J. NeuroEngineering & Rehabilitation*, vol. 17, Art. 14, 2020, doi: https://doi.org/10.1186/s12984-020-0644-3

Compares 3 different compensation methods which we can use to help our decision for our control approach

Literature Review

- [38] C. Hill *et al.*, “Gravity Compensation of an Exoskeleton Joint Using Constant-Force Springs,” in *Proc. IEEE Int. Conf. Rehabilitation Robotics (ICORR)*, 2019, pp. 1032–1037, doi: https://doi.org/10.1109/ICORR.2019.8779422

  Constant-force springs used to lower torque needs in an exoskeleton joint.

- [39] J. Vantilt *et al.*, “Model-based control for exoskeletons with series elastic actuators,” *J. NeuroEngineering & Rehabilitation*, vol. 16, Art. 100, 2019, doi: https://doi.org/10.1186/s12984-019-0526-8

  Shows kinematic/dynamics modelling steps needed for final kinematic model of our design.

- [40] ASTM International, *F3323-21: Standard Terminology for Exoskeletons and Exosuits*, 2021. Available: https://www.astm.org/f3323-21.html

  Official definition of (exoskeleton, exosuit assist level and more) for keeping our slides consistent.

- [41] ISO, *ISO 13485:2016 - Medical devices — Quality management systems — Requirements for regulatory purposes*, 2016. [Online]. Available: https://www.iso.org/standard/59752.html

  Regulations of medical devices to use for our design.

Torque with Wrist Connection

- How much torque is required to hold arm forward from shoulder if the connection point is at the wrist?

- Upper Arm
  - L_{UA}= 0.3m
  - m_{UA} = 2.25 kg
- Forearm
  - L_F = 0.27m
  - m_f = 1.73 kg
- Hand
  - L_H = 0.19m
  - m_H = 0.49 kg

Gravitational Torque (τ) = Mass (m) * Acceleration due to Gravity (g) * Radius (r)

$$\tau_w = g\left(m_{UA}\left(L_F + \frac{L_{UA}}{2}\right) + m_F\left(\frac{L_F}{2}\right) + m_H\left(\frac{L_H}{2}\right)\right) \quad (1)$$

$$\tau_w = 12.02\ N \cdot m$$

Torque with Elbow Connection

- How much torque is required to hold arm forward from shoulder if the connection point is at the elbow?
- Torque Summation: $\tau_{el} = g\left(m_F \cdot \frac{L_F}{2} + m_H\left(L_F + \frac{L_H}{2}\right)\right)$ $= 4.24\ N \cdot m$

(2)

- Takeaway: If the connection point is at the elbow, it will require less torque to extend the arm using a connection at the elbow compared to the wrist which will be important when choosing between two initial prototypes.

Spring Force at the Wrist Required to Negate Weight of the Arm

$$F_{req} = \frac{\tau}{r \times \sin(\theta)} \qquad (3)$$

$$F_{req} = \frac{12.02}{0.76 \times \sin(45)}$$

$$F_{req} = 22.37N$$

Therefore we should aim for A

spring with a force larger than

25N giving a ten percent tolerance

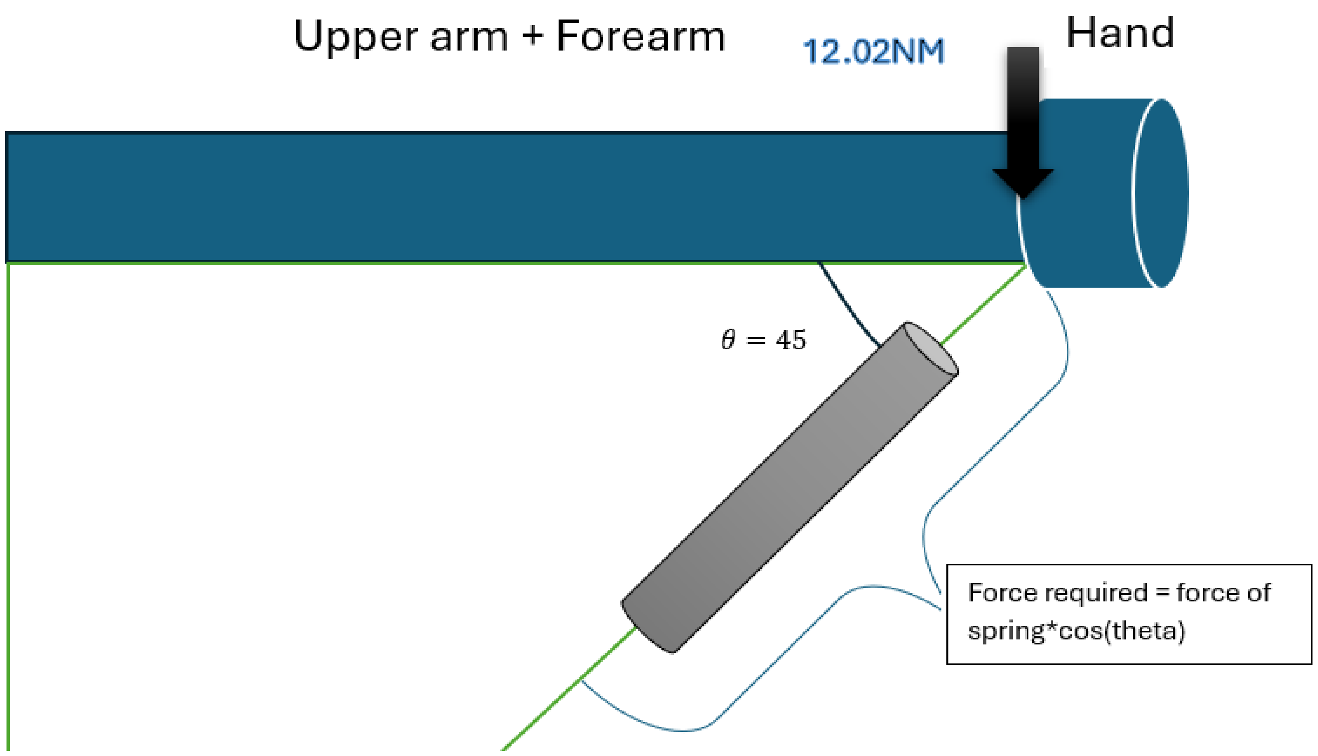

Figure 4

Misalignment Forces

Why misalignment forces are important?
Transitional Equation: $F_{miss} = k\,\Delta x$ (3)   => 2000 · 0.008 = 16 N
The user feels 16 N extra force pressing against the arm due to misalignment.
If tolerance target is ≤15 N → this design is borderline.

Angular: $M_{miss} = F \cdot r \cdot sin(\Delta x)$ (4) => $40 \cdot 0.05 \cdot \sin(5°)$ = 0.174 N/m
A small torque (~0.17 N·m) is felt at the wrist, which could cause strain over long usage.

Coupling: $\tau_{mis} = J_{err} \cdot F_{int}$ (4) => $0.015 \cdot 30$=0.45 N/m
Parasitic torque of ~0.45 N·m is transmitted to the shoulder joint even though the exo was “helping"

Energy cost: $E_{extra} = \int F_{mis} \cdot V\,dt$ (5) => $16 \cdot 0.15 \cdot 60$ = 144 J
The user expends 144 J extra energy in one minute — equivalent to lifting 14 kg up by 1 m. Over rehab sessions, this is a noticeable fatigue factor.

Bending Stress of Varying Beam Geometries in Link 2

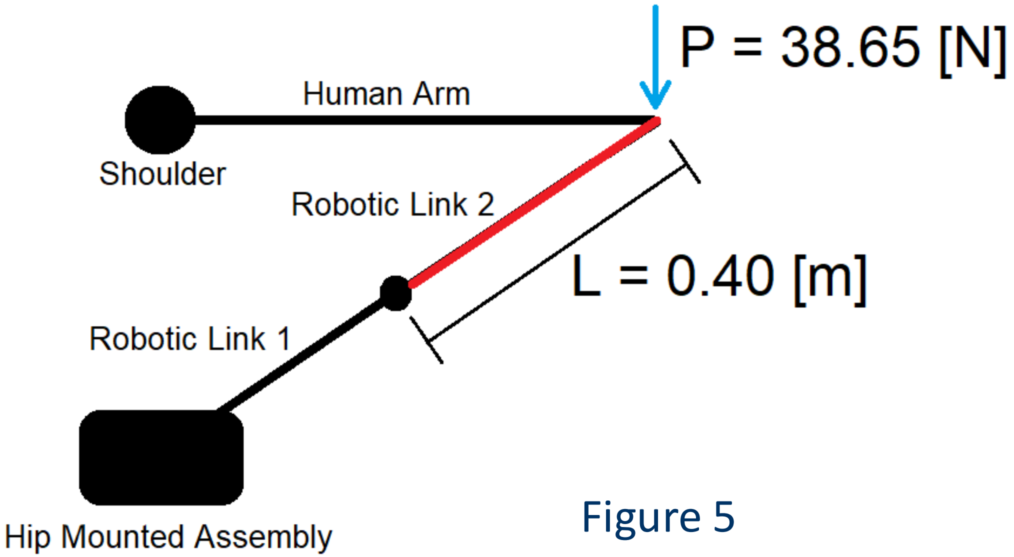

Figure 5

- $\sigma_{\max_c} = \frac{M_{max} \cdot c}{I_c} = 15.4\,[MPa] \leq 80.33\,[MPa]$ (6)
- $\sigma_{\max_s} = \frac{M_{max} \cdot c}{I_s} = 69.4\,[MPa] \leq 80.33\,[MPa]$ (7)
- $\sigma_{allowable} = \frac{\sigma_{yeild}}{FoS} = 80.33\,[MPa]$ (8)

- What will be the bending stress in linkage 2 given different geometries?
  - Compared 2 geometries; a circular tube (c), and a square tube (s).
  - Material was chosen as 6061-T6 Aluminum due to its lightweight and higher strength properties.
  - Both geometries are strong enough for the application, however, the circular tube is over 4 times stronger.
  - Achieving a FoS > 3 is only achievable with the circular cross-sectional geometry.
  - See Appendix A for assumptions, technical variables, and supporting calculations

Budget

- Funding from W.L Gore: $4000
- NAU 5% processing fee: -$200
- Fundraising (at least %10): $400
- Total Est. Budget: $4200

- We have an estimate of $5750 for total cost of possible items. The team will have a remaining balance of -$1550. We will need to fundraise more to begin prototyping.

Expenses		
Category	Items(s)	Cost
Tools and materials:	3D printer Parts	$200
	3D printer Filament	$50
Manufacturing:		$600
Parts:	Motors	$1000
	Battery	$400
	Metal Rods	$500
	Coverings	$500
Prototyping:	1st	$1500
	2nd	$1000
TOTAL:		$5750

Fundraising

- Need to accumulate 10% of the $4,000 budget for a minimum of $400 total
- In talks with multiple companies regarding sponsorships, services, or cash donations
- We plan to fundraise the entire 10% on or before week 8, or the testing of prototype 1

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Schedule

Plan duration | Actual Start | Completed | Beyond Completion

	August				September				October				November				December			
				wk1	wk2	wk3	wk4	wk5	wk6	wk7	wk8	wk9	wk10	wk11	wk12	wk13	wk14	wk15	wk16	
Requirements/ research																				
Equations																				
Presentation 1																				
Conceptual designs																				
Fundraising																				
Presentation 2																				
Begin modelling																				
Testing prototype 1																				
Presentation 3																				
1st Prototype Demo																				
Begin prototype 2																				
Testing for Prototype 2																				
2nd Prototype Demo																				

Thank you
And
Any Questions?

Appendix

- [1] S. D. Gasperina *et al.*, "AGREE: A compliant-controlled upper-limb exoskeleton for physical rehabilitation of neurological patients," *IEEE Transactions on Medical Robotics and Bionics*, pp. 1–1, 2023, doi: https://doi.org/10.1109/tmrb.2023.3239888.
- [2] R. Soltani-Zarrin, Amin Zeiaee, A. Eib, Reza Langari, N. Robson, and Reza Langari, "TAMU CLEVERarm: A novel exoskeleton for rehabilitation of upper limb impairments," Nov. 2017, doi: https://doi.org/10.1109/werob.2017.8383844.
- [3] I. M. Alguacil-Diego *et al.*, "Validation of a Hybrid Exoskeleton for Upper Limb Rehabilitation. A Preliminary Study," *Sensors*, vol. 21, no. 21, pp. 7342–7342, Nov. 2021, doi: https://doi.org/10.3390/s21217342.

Appendix A

Broad Assumptions:

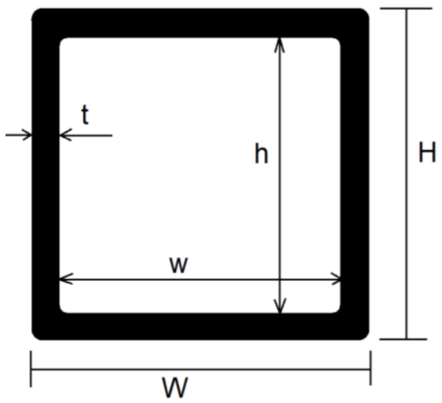
- Average Human male arm mass = 1.89 [kg]
  - Average total body mass of 75 [kg]
  - Forearm Mass + Wrist Mass = 2.52% of Total Body Mass
- Arm is fully outstretched in front of the body
- Linkage at 45 degrees
- Treat connection to link 1 as fixed
- Robotic linkage is split into 2 parts of equal length (0.40 [m])
- Deflection due to Axial compression is negligible

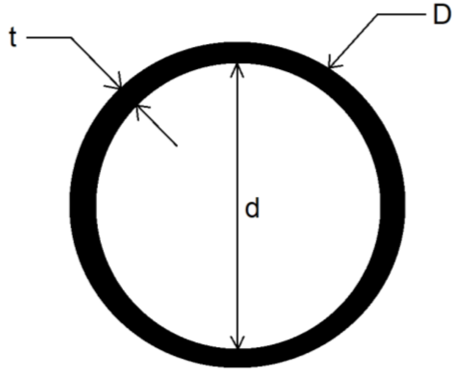
Technical Variables:

- Material: 6061-T6 Aluminum
- $FoS = 3$
- $g = 9.81 \left[\frac{m}{s^2}\right]$
- $m_{total} = 75\ [kg]$
- $\sigma_{yeild} = 241\ [MPa]$
- $a = 0.0525$
- $E = 6.89 \cdot 10^{10}\ [Pa]$ Elastic Modulus
- $L = 0.40\ [m]$

- $c = 0.015$
- $t = .003\ [m]$
- $\theta = 45^{\circ}$
- $D = 0.254\ [m]$
- $d = 0.0251\ [m]$
- $H = 0.0254\ [m]$
- $W = 0.0254\ [m]$
- $h = 0.0251\ [m]$
- $w = 0.0251\ [m]$

Geometries:





Calculations:

- $c = \frac{D}{2} = \frac{H}{2} = 0.0127\ [m]$
- $m_{arm} = m_{total} \cdot a = 3.94\ [kg]$
- $P = m_{arm} \cdot g = 38.65\ [N]$
- $P_{\perp} = P \cdot \cos\theta = 27.33\ [N]$
- $M_{max} = P_{\perp} L = 10.93\ [Nm]$
- $I_c = \frac{\pi}{64}(D^4 - d^4) = 9 \cdot 10^{-6}\ [m^4]$
- $I_s = \frac{HW^3 - hw^3}{12} = 2 \cdot 10^{6}\ [m^4]$
- $\delta_{\perp c} = \frac{P_{\perp} L^3}{3EI_c} = .00094\ [mm]$
- $\delta_{\perp s} = \frac{P_{\perp} L^3}{3EI_s} = .004\ [mm]$